Spiral Galaxies When Disks Dominate their Halos (using Arm Pitches and Rotation Curves)

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Abstract

Spiral galaxies' disk dynamics are usually considered to be dominated by more massive dark matter halos, particularly if their rotation curves, *V*, are flat relative to radius, *r*. Disk surface mass density as a function of *r* and its importance relative to its halo is derived from the density wave arms' winding pitch *i* and variable *V*. Results are obtained for examples with flat *V* and log arms of multiplicity *m*. The density wave disk importance relative to the halo is the ratio $F_{D/H}=F_D$ / F_H where $F_D= 4 \tan i /m$ and $F_H=1- F_D$. For the tightly wound NGC 7217 with small *i*, the disk is unimportant relative to the halo, fitting expectations. For less tight M51 the disk is comparable to the halo, reasonable. But for the looser M100, the density wave disk is ≈ 2 times the halo. For the still looser M101 the disk is ≈ 3 times the halo. For the loosest, NGGC 3198, the disk appears to totally dominate the halo. This sequence of progressively more important disks challenges the commonly accepted preeminence of spiral galaxies' halos over their disks.

Introduction

IT IS COMMONLY ACCEPTED that spiral (S) type galaxies' disk dynamics are usually dominated by more massive dark matter halos, particularly if their rotation curves (*V)* relative to radius (*r*) are flat well beyond the visible disk. As reviewed by Byrd and Valtonen (2020), this was first detected at 21 cm, well beyond the M31 visible disk as shown in Figure 1 (Whitehurst and Roberts 1972 and Roberts and Whitehurst 1975). Later, Bosma (1978) and Rubin et al (1978) observed extended flat curves optically for a number of Sa-Sc galaxies supporting the interpretation that the outer flat *V* is generated by a more spherical halo which dominates the dynamics. It is argued that a disk dense enough to solely create the flat curve would be unstable.

After initial acceptance there were some doubts as reviewed by Pfenniger (2000) who also found that simulation models containing certain massive disk components can be well behaved. Here we show via observation and dynamical theory that some S galaxies are disk dominated rather than halo dominated over their arm regions (even with flat rotation curves).

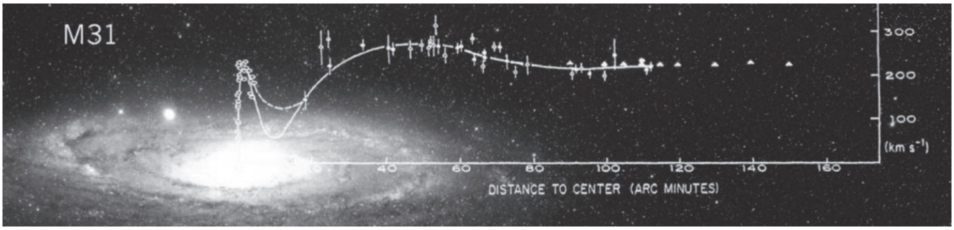

Figure 1. Superposed rotation curves on M31. Inner visual curve from Vera Rubin and 21 cm from Whitehurst and Roberts (1972) and Roberts and Whitehurst (1975). Beyond the visible disk, the flat 21 cm curve extends at least 34 kpc! Figure permission from Mort Roberts.

Arm Properties

Danver (1942) fit the arms in S galaxies and found logarithmic (log) arms in which the pitch (acute angle i with the circular arc) is constant as the arm r increases. Kennicutt (1981) found that log arms are very common. More sophisticated fittings have since been carried out with the arm pattern being the superposition of individual logarithmic spiral components by Considere and Athanassoula (1988). They found that the two-armed component is everywhere dominant.

Grand design spirals show a range of arm pitches. NGC 7217 has a tightly wound arm pair with, i= 4.8° (Buta *et al*. 1995, Figure 10) and a flat V (Kalnajs 1983). Elmegreen *et al*. (1989) fit log spirals to arm patterns and obtained 16°, 18°, and 15° respectively in the disks of M81 (NGC3031), M100 (NGC4321) and M51 (NGC5194). Honig and Reid, (2015, Figure 5) examined HII regions in M101 (NGC4321) to trace a two arm pattern whose i =20°. Going steeper, NGC 3198's arms have i = 30.0± 6.7° (Ferrarese 2002). Again, all have flat rotation curves over their density wave regions.

M51 has log arms in the main part of the disk. However, the arms near the disk edge are instead winding tidal arms due to a past companion passage (Howard and Byrd 1990 or Byrd 1995). We will discuss just type S galaxies, not strongly barred SB galaxies. In SBs arms or rings may be a forced periodic orbit pattern imposed by the bar. This pattern can also be used to estimate disk surface mass density, μ_D and mass-to-light ratio (Byrd et al. 2006).

Notation and Definitions

We derive the μ_D for the disks of spiral galaxies as a function of r as well as disk mass importance relative to its halo using the spiral arm's i and

V. The disk μ_D and disk/halo calculations are for disk regions where the arms are a classical grand design density wave pattern as proposed and developed by Lindblad (1960), Lin and Shu (1966) and Lin, Yuan, and Shu (1969).

At each point in the arm r typically increases with increasing azimuthal angle, φ, in the disk. The rotation curve is V. The orbital angular rate ($\Omega = V/r$) declines with r. For classical density waves, the arms turn with angular rate pattern speed, $\Omega_{p.} = V / r_{CR}$ where r_{CR} is the CR radius. The arm multiplicity is denoted by m for the single m=1 or the m = 2 arm pair etc. The wave number, k, wavelength, λ, and, i, have definitions given by:

$$\left|k\right| = \frac{m}{r \tan i} \tag{1}$$

$$\tan i = \frac{\Delta r}{\Delta \phi r} = \frac{2\pi}{\lambda}. \tag{2}$$

Disk Surface Density Calculation

We now calculate μ_D. The dispersion relation provides a condition for stability determined by the competing self-gravity of μ_D, velocity dispersion, a, and orbital angular rate, $\Omega = V/r$, and the shear. The density wave dispersion relation is given by:

$$S = \left(\omega - m\Omega\right)^2 = \kappa^2 + k^2 a^2 - 2\pi G\left|k\right|\mu \tag{3}$$

where m is arm multiplicity (Shu 1982). Here the epicyclic frequency squared is

$$\kappa^2 = r^3 \frac{d}{dr}\left(r^4\Omega^2\right) = 4\Omega^2 + 2r\Omega\frac{d\Omega}{dr}. \tag{4}$$

The wave frequency is

$$\omega = m\Omega_P, \tag{5}$$

where the arm pattern speed is Ω_p.

A stable oscillation condition for a density wave ω is reached where the minimum disk velocity dispersion is

$$a_{\min} = \frac{\pi G \mu_D}{\kappa} \tag{6}$$

as is given in Shu (1982) for the axisymmetric case.

What about non-axisymmetric case? By using the swing amplification approach of Toomre (1981) for S, Byrd (1995's eqn. 12b) obtains close to the Equation (6) expression for tilted winding arms. S can be described as the spring constant for oscillation of stars relative to the arm. If there is no μ_D, then pure epicyclic oscillation is left. If a stellar disk with velocity dispersion less than a minimum a_{min} is perturbed, small perturbation arms will wind up to where S is less than zero. Then the arm stars' oscillation is unstable to collapse "relaxation" so the velocity dispersion increases to a_{min} for S of zero. We add growth from an initial perturbation to collapse relaxation as part of dispersion's origin in spiral arms by Merrifield *et al* (2001).

For $a = a_{min}$ S is at a minimum resulting in a longer term density wave. The observed values of m, i, the angular rate, $\Omega(r)$, and the pattern speed, Ω_p , permit calculation of μ_D . We substitute Equation (6) into Equation (3) to get a quadratic equation for μ_D. There are two possible values[1] for a given $(\Omega - \Omega_p)^2$. We choose

$$\mu_D = \left(\frac{\kappa^2}{\pi G}\right)\left(\frac{r \tan i_m}{m}\right)\left[1 + \frac{m(\Omega - \Omega_P)}{\kappa}\right]. \tag{7}$$

because inside the square bracket, when Ω declines with respect to radius, the second term is positive inside CR and negative outside. Thus the surface density declines with radius. The equivalent equation is derived by Shu (1982).

At CR, where $\Omega = \Omega_p$, the second term in the square bracket is zero, so Equation (7) is close to that derived for winding arms at CR by Byrd (1995):

$$\mu_D = \left(\frac{e\kappa^2}{2\pi G}\right)\left(\frac{r \tan i_m}{m}\right). \tag{8}$$

The above derivation of Equation (7) uses only the i of the spiral arms and the disk V which can vary with radius. While we will assume a flat V and constant i for the latter parts of this article, we emphasize that Equation (7) has been applied to NGC 4622 which definitely has a non-flat rotation curve (Byrd and Howard 2019).

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[1] A quadratic equation typically has two solutions.

Disk Importance Relative to Halo

Now we expand and generalize Equation (7) by adding the concept of a halo and removing the need to know the CR. We derive the importance of an S galaxy's disk mass relative to its halo using its observed constant *i* and flat *V*. We do the calculation for examples of S galaxies with flat rotation curves over their density wave regions.

By substituting $\Omega = V/r$ and $\kappa^2 = 2\ V^2/r^2$ into Equation (7), the disk surface density for a flat rotation curve is

$$\mu_{D,V} = 2\left(\frac{V^2}{\pi G}\right)\left(\frac{\tan i_m}{mr}\right)\left[1+\frac{m\left(1-r/r_{CR}\right)}{\sqrt{2}}\right]. \tag{9}$$

We will compare the disk to a spherical flat rotation curve halo whose mass interior to a radius is

$$M_{H<r} = \left(\frac{V^2}{G}\right)r. \tag{10}$$

The inner and outer Lindblad resonance radii of the arms are,

$$r_{ILR} = r_{CR}\left(1-m/\sqrt{2}\right) \text{ and } r_{OLR} = r_{CR}\left(1+m/\sqrt{2}\right). \tag{11}$$

Donner and Thomasson (1994) demonstrated that arms reach these density wave radial limits in their disk galaxy simulations.

Substituting, simplifying, and integrating Equation (9), we get the disk mass, $M_{D,\,V}$ between the *ILR* and *OLR* of the density wave,

$$M_{D,V} = \int 2\pi\mu r dr = \left(\frac{4V^2}{G}\right)\left(\frac{\tan i_m}{mr}\right)\left(1+\frac{m}{\sqrt{2}}\right)\left(r_{OLR}-r_{ILR}\right) - m\frac{\left(r_{OLR}-r_{ILR}\right)\left(r_{OLR}+r_{ILR}\right)}{2\sqrt{2}c_{CR}}. \tag{12}$$

The halo mass between the *ILR* and *OLR* is

$$M_{100\%,H} = \left(\frac{V^2}{G}\right)\left(r_{OLR}-r_{ILR}\right). \tag{13}$$

We divide the actual disk mass, $M_{D,V}$, by $M_{100\%,H}$ to obtain the mass ratio, F_D , of the actual disk to a full 100% halo for the density wave region as

$$F_D = 4\tan i/m. \tag{14}$$

The halo minus disk value is $F_H = 1 - F_D$. The disk compared to the actual halo is $F_{DH} = F_D / F_H$.

Using Equation (14), for our example galaxies, the 4.8° pitch of NGC 7217's arm pair indicates $F_D \approx 0.13$, a disk much less than the halo. The 15° pitch of the M51 log arm pair indicates $F_D \approx 0.54$, a disk comparable to the halo. Figure 2 compares a simulation with $F_D \approx 0.5$ to a photo of M51. While the simulation does well to simulate the density wave, effects of particle statistics and softening need to be taken into account for more tightly wound cases. Softening can be equivalent to as much as two times a_{min} in simulations like Figure 2 (Byrd 1995).

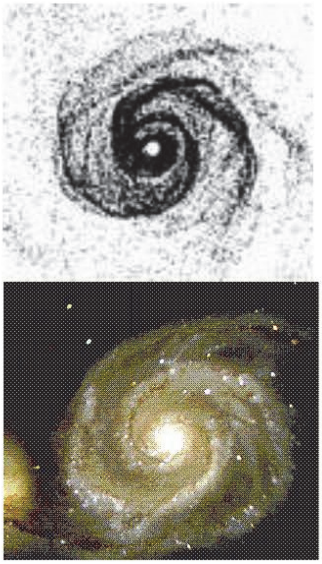

Figure 2. Simulation with $F_D = 0.5$ compared to an image of the M51 whose log arm pair whose 15° pitch indicates $F_D \approx 0.54$. Image courtesy of William Keel, University of Alabama. The companion on the lower left is far behind the disk plane, and perturbed the disk in the distant past (Byrd and Howard 1990).

The disk can be more important than the halo. For M100 the 18° pitch indicates $F_D \approx 0.63$, two times larger than the halo. The 20° pitch of M101 indicates by Equation (13) that $F_D \approx 0.73$, three times the halo. For our steepest example, NGC 3198, the 30.0± 6.7° pitch gives $F_D \approx 1.15$! Of course, F_D cannot be greater than one. The lower 23.3° value gives a more physical $F_D \approx 0.86$, still six times the role of the halo. See Figure 3 for an image of this low-halo galaxy. These last two spirals are in a sequence of progressively more important disks in our examples, challenging the overall preeminence of the halo over the disk. The observation of few if any Sc galaxies steeper than NGC 3198's pitch is explained.

Figure 3. NGC 3198, a no-halo disk galaxy?

Conclusions

We derive how to calculate the surface mass density of the disks of spiral galaxies as a function of radius as well as the disk mass importance relative to its halo. The derivation uses only the degree of arm winding tightness or looseness along with the rotation curve for the estimates. We obtain equations both for the case of a non-linear rotation curve as well as for the common flat one.

The disk to the halo ratio is obtained over the density wave arm pattern region for example spirals with flat rotation curves. For the tightly wound NGC7217 we find the disk is much less important than the halo over the spiral arm region. For the intermediately wound M51 we find the disk to be comparable to the halo in mass over the disk arm region. For the looser armed M100 we find the disk to be twice the halo. For the loosest M101 the disk evidently dominates by about a factor of three.

For our steepest example, NGGC 3198, the disk appears to totally dominate the halo. Even with the approximations, this example sequence of progressively more important disks challenges the commonly accepted preeminence of spiral galaxies' halos over their disks.

Our results provide a starting point for more informed investigations *e.g.* of halo versus disk dark matter for different galaxy Hubble types. In another example, it has been found observationally that there is an anti-correlation between the central black hole mass and arm inclination in spiral galaxies. Smaller arm pitches are correlated with more massive central black holes (Seiger *et al*. 2008, Berrier *et al*. 2013a, b, Savchenko and Reshetnikov 2013). A physical connection of low disk mass (relative to halo) with central black holes is suggested.

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Bio

Gene Byrd (B.S Texas A&M Univ. 1968; PhD 1974 the Univ. of Texas) is a Professor of Astronomy (emeritus) at the Univ. of Alabama. He studies the dynamics of galaxies, discovering the pattern in NGC4622, which, counter-intuitively, has inner and outer spiral arms winding in opposite directions See https://www.researchgate.net/profile/Gene_Byrd2 .

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